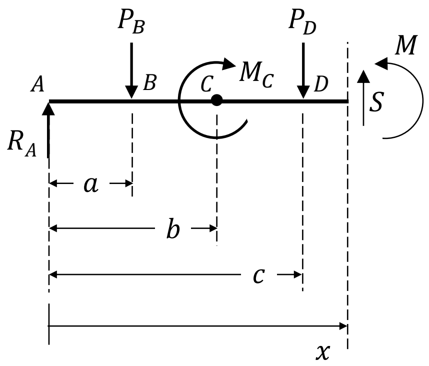
2020-2021 In-Class Test Solutions

1.

A. $EI\frac{d^2y}{dx^2} = R_A x + M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle$

SOLUTION 1

Sectioning the beam after the last discontinuity (taking origin at left hand side of the beam) and drawing a Free Body Diagram of left-hand side of section:



Taking moments about the section position (and applying Macauley's convention):

$$M + P_B\langle x-a\rangle + P_D\langle x-c\rangle = R_A x + M_C\langle x-b\rangle^0$$

$$\therefore M = R_A x + M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle$$

Substituting this into the deflection of beams equation:

$$\boldsymbol{EI\frac{d^2y}{dx^2} = R_A x + M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle}$$

2.

C. $\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_A x^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2} + A\right)$

SOLUTION 2

Integrating $EI\frac{d^2y}{dx^2} = R_A x + M_C\langle x-b\rangle^0 - P_B\langle x-a\rangle - P_D\langle x-c\rangle$ with respect to x gives:

$$EI\frac{dy}{dx} = \frac{R_A x^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2} + A$$

Rearranging this for $\frac{dy}{dx}$:

$$\boldsymbol{\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_A x^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2} + A\right)}$$

3.

E. $y = \frac{1}{EI}\left(\frac{R_A x^3}{6} + \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6} + Ax + B\right)$

SOLUTION 3

Integrating $\frac{dy}{dx} = \frac{1}{EI}\left(\frac{R_A x^2}{2} + M_C\langle x-b\rangle - \frac{P_B\langle x-a\rangle^2}{2} - \frac{P_D\langle x-c\rangle^2}{2} + A\right)$ with respect to x gives:

$$\boldsymbol{y = \frac{1}{EI}\left(\frac{R_A x^3}{6} + \frac{M_C\langle x-b\rangle^2}{2} - \frac{P_B\langle x-a\rangle^3}{6} - \frac{P_D\langle x-c\rangle^3}{6} + Ax + B\right)}$$

4.

B. AB

5.

E. No

SOLUTION 5

$$\tau = \frac{Tr}{J} = \frac{32 \times 1100 \times 19 \times 10^{-3}}{\pi \times (19 \times 10^{-3})^4} = 102 \text{ MPa}$$

$$\sigma_{BM} = \frac{Mr}{I} = \frac{64 \times 1300 \times 19 \times 10^{-3}}{\pi \times (19 \times 10^{-3})^4} = 241 \text{ MPa}$$

$$C = \frac{\sigma_{BM}}{2} = \frac{241}{2} = 120.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{z\theta}^2} = \sqrt{\left(\frac{241}{2}\right)^2 + 102^2} = 158 \text{ MPa}$$

$$\sigma_1 = C + R = 120.5 + 158.2 = 278.7 \text{ MPa}$$

$$\sigma_2 = C - R = 120.5 - 158.2 = -37.4 \text{ MPa}$$

$$\mathbf{299 < \boldsymbol{\sigma_y} = 300\ MPa}$$

6.

A. Yes

SOLUTION 6

Behaviour is assumed to be all elastic and therefore:

$$\frac{M_y}{I} = \frac{\sigma_y}{y}$$

where M_y is the moment required to cause yielding.

First yield will occur at $y = \pm\frac{d}{2}$, i.e. at the top and bottom edges:

$$\therefore M_y = \frac{\sigma_y \times I}{y} = \frac{\sigma_y \times \left(\frac{bd^3}{12}\right)}{\frac{d}{2}} = \frac{205 \times \left(\frac{150^4}{12}\right)}{\frac{150}{2}} = 115{,}312{,}500\ Nmm = 115.3\ kNm$$

Since $M > M_y$, yielding does occur.

7.

E. Linear hardening

8.

C. 6 mm

SOLUTION 8

For an internally pressurized cylinder:

$$\sigma_1 = \sigma_\theta = \frac{pt}{t}$$

	A	B	**C**	D	E
t	1.00E-02	8.00E-03	**6.00E-03**	4.00E-03	2.00E-03
r	0.5	0.5	**0.5**	0.5	0.5
p	2.00E+06	2.00E+06	**2.00E+06**	2.00E+06	2.00E+06
sigma_a	5.00E+07	6.25E+07	**8.33E+07**	1.25E+08	2.50E+08
sigma_th	1.00E+08	1.25E+08	**1.67E+08**	2.50E+08	5.00E+08

9.

D. 3.14×10^{-4} m

SOLUTION 9

$$\Delta L = L\alpha\Delta T$$

$$\Delta L = 1.9 \times 11 \times 10^{-6} \times 15 = \mathbf{3.14 \times 10^{-4}\ m}$$

10.

C. 1.15 MPa

SOLUTION 10

von Mises yield criterion states yield occurs when:

$$\sigma_{sf} = \frac{215}{1.5} = 143.3 \text{ MPa}$$

$$\sigma_y^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

For an internally pressurised cylinder

$$\sigma_1 = \sigma_\theta = \frac{pr}{t}$$

$$\sigma_2 = \sigma_A = \frac{pr}{2t}$$

Substituting in:

$$\sigma_y^2 = \frac{p^2r^2}{t^2} - \frac{p^2r^2}{2t^2} + \frac{p^2r^2}{4t^2}$$

$$\sigma_y^2 = \frac{3}{4}\frac{p^2r^2}{t^2}$$

$$p = \sqrt{\frac{4}{3}}\frac{\sigma_y\, t}{r}$$

$$p = \sqrt{\frac{4}{3}}\frac{143.3 \times 10^6 \times 4 \times 10^{-3}}{0.575} = \mathbf{1.15\ MPa}$$

11.

C. The Soberberg line is a better option than the Goodman line or the Gerber curve

12.

E. 18 MPa

SOLUTION 12

$$F_{copper} = \frac{1000 + \alpha \Delta T E_{al} A}{1 + \frac{E_{al}}{E_{copper}}} = \frac{1000 + 23 \times 10^{-6} \times 5 \times 70 \times 10^{9} \times 50 \times 10^{-6}}{1 + \frac{70 \times 10^{9}}{128 \times 10^{9}}} = 907\ \text{N}$$

$$\sigma_{copper} = \frac{F_{copper}}{A} = \frac{907}{50 \times 10^{-6}} = 18.14 \times 10^{6}\text{Pa} = \mathbf{18\ MPa}$$

13.

D. D

SOLUTION 13

For cross-section A:

$$I = \frac{bd^3}{12} = \frac{300 \times 150^3}{12} = 84{,}375{,}000 \text{ mm}^4$$

For cross-section B:

$$I = \frac{\pi\left({D_o}^4 - {D_i}^4\right)}{64} = \frac{\pi \times (300^4 - 250^4)}{64} = 205{,}860{,}221.7 \text{ mm}^4$$

For cross-section C:

$$I = \frac{b_o {d_o}^3 - b_i {d_i}^3}{12} = \frac{250^4 - 100^4}{12} = 317{,}187{,}500 \text{ mm}^4$$

For cross-section D:

$$I = \frac{bd^3}{12} = \frac{150 \times 300^3}{12} = \mathbf{337{,}500{,}000\ mm^4}$$

For cross-section E:

$$I = \frac{\pi D^4}{64} = \frac{\pi \times 250^4}{64} = 191{,}747{,}598.5 \text{ mm}^4$$

Cross-section D has the largest 2nd moment of area.

14.

B. 252.3 mm

SOLUTION 14

2nd moment of area of a solid rectangular cross-section:

$$I = \frac{bd^3}{12}$$

Where, for a square, $b = d$,

$$\therefore I = \frac{b^4}{12}$$

$$\therefore b = \sqrt[4]{12I}$$

Substituting in the largest 2nd moment of area from Q7 ($337{,}500{,}000\ \text{mm}^4$):

$$D = \sqrt[4]{12 \times 337{,}500{,}000} = \mathbf{252.3\ mm}$$

15.

D. $T = 180\ \text{Nm}, P = 200\ \text{N}, M = 220\ \text{Nm}$

SOLUTION 15

$$\tau = \frac{Tr}{J}$$

$$\sigma_{BM} = \frac{Mr}{I}$$

$$\sigma_A = \frac{P}{A}$$

$$\sigma_Z = \sigma_{BM} + \sigma_A$$

$$C = \frac{\sigma_Z + \sigma_\theta}{2} = \frac{\sigma_Z}{2}$$

$$R = \sqrt{\left(\frac{\sigma_Z - \sigma_\theta}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{\sigma_Z}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = C + R$$

	A	B	C	**D**	E
d	5.00E-02	5.00E-02	5.00E-02	**5.00E-02**	5.00E-02
T	200	200	220	**180**	350
P	200	220	180	**200**	100
M	200	180	200	**220**	100
sigma_1	1.98E+07	1.84E+07	2.03E+07	**2.06E+07**	1.89E+07

16.

C. 76.2 MPa

SOLUTION 16

Given $\sigma_x = 23 \text{ MPa}$, $\sigma_y = 73 \text{ MPa}$ and $\tau_{xy} = 13 \text{ MPa}$

$$C = \frac{\sigma_x + \sigma_y}{2} = \frac{23 + 73}{2} = 48 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{z\theta}^2} = \sqrt{\left(\frac{23 - 73}{2}\right)^2 + 13^2} = 28.18 \text{ MPa}$$

$$\sigma_1 = C + R = 48 + 28.2 = \mathbf{76.2\ MPa}$$

17.

E. 26.94 mm

SOLUTION 17

$$K_I = 2.75\sigma\sqrt{\pi a}$$

$$\therefore K_{Ic} = 2.75 \times \frac{2}{3}\sigma_y\sqrt{\pi a_{cr}}$$

where

$$K_{Ic} = 120 \text{ MPa}\sqrt{\text{m}}$$

and

$$\sigma_y = 225 \text{ MPa}$$

Therefore,

$$120 = 2.75 \times \frac{2}{3} \times 225 \times \sqrt{\pi a_{cr}}$$

$$\therefore a_{cr} = \left(\frac{120}{2.75 \times \frac{2}{3} \times 225}\right)^2 \times \frac{1}{\pi} = 0.02694 \text{ m} = \mathbf{26.94\ mm}$$

18.

B. Decreasing R-ratio

SOLUTION 18

A decreased R-ratio typically means a decreased mean stress which slows down crack growth.

19.

A. 30.6 mm

SOLUTION 19

vM criteria for plane stress, yield occurs when:

$$\sigma_y^2 = \sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2$$

As shaft is under pure torque, Mohr's circle is centred on the origin and $|\sigma_1| = |\sigma_2| = k = \tau_{max} = \frac{Tr}{J}$

Substituting in the expression for τ_{max} into the vM equation above and rearranging for r gives:

$$r = \sqrt[3]{\frac{2T}{\sigma_y \pi}} = \sqrt[3]{\frac{2 \times 18000}{400 \times 10^6 \times \pi}} = 0.0306 \text{ m} = \mathbf{30.6\ mm}$$

20.

B. −94.8 MPa

SOLUTION 20

$$\frac{F}{A} = \frac{F_{init}}{A} - E\alpha\Delta T = \frac{2100}{75 \times 10^{-6}} - 70 \times 10^9 \times 22 \times 10^{-6} \times 80 = -9.48 \times 10^7 \text{Pa} = \mathbf{-94.8\ MPa}$$